

The average time for the moon to circle the earth
(Synodic month) as it circles the sun is:

$$= 29.53059 \text{ days/month}$$

$$= 29\text{d } 12 \text{ h } 44\text{m } 03\text{s/month}$$

The average year is

$$= \left(29.53059 \frac{\text{days}}{\text{month}}\right)(12 \text{ months})$$

$$= 354.367 \text{ days}$$

YEAR 1

Since years can have 353 days, 354 days or 355 days, the first year will have 354 because that number is closest to the average year length.

The greatest variation from the average: $\approx 6.5 \text{ hrs}$

$$\begin{aligned} \text{Average length of month + greatest variation} &= 29\text{d } 12 \text{ h } 44\text{m } 03\text{s/month} + 6 \text{ hrs } 30 \text{ m} \\ \text{(longest month)} &= 29\text{d } \mathbf{19 \text{ h } 14\text{m}} \text{ } 03\text{s/month} \end{aligned}$$

$$\begin{aligned} \text{Average length of month – greatest variation} &= 29\text{d } 12 \text{ h } 44\text{m } 03\text{s/month} – 6 \text{ hrs } 30 \text{ m} \\ \text{(shortest month)} &= 29\text{d } \mathbf{06 \text{ h } 14\text{m}} \text{ } 03\text{s/month} \end{aligned}$$

The number of days in the moon cycle cannot be less than 29 or greater than 30.

YEAR 2

Length for the second year can be 353 days, 354 days, or 355 days.

$$\begin{aligned} \text{If it is } \mathbf{355 \text{ days}} \text{ – then need to add } 354 \text{ days from year 1 and} &= 354 \text{ days} + 355 \text{ days} \\ \mathbf{355 \text{ days}} \text{ from year 2} &= 709 \text{ days in year 1, 2} \end{aligned}$$

The number of months in the first 2 years can be found as follows:

$$\begin{aligned} \text{months} &= \frac{\text{total days}}{\frac{\text{days}}{\text{month}}} \\ &= \frac{709 \text{ days}}{29.53059 \frac{\text{days}}{\text{month}}} \\ &= 24.009 \text{ months} \end{aligned}$$

If it is **354 days** – then need to add 354 days from year 1 and 354 days from year 2 = 354 days + 354 days
 = 708 days in year 1, 2

The number of months in the first 2 years can be found as follows:

$$\begin{aligned} \text{months} &= \frac{\text{total days}}{\frac{\text{days}}{\text{month}}} \\ &= \frac{708 \text{ days}}{29.53059 \frac{\text{days}}{\text{month}}} \\ &= 23.975 \text{ months} \end{aligned}$$

If it is **353 days** – then need to add 354 days from year 1 and 353 days from year 2 = 354 days + 353 days
 = 707 days in year 1, 2

The number of months in the first 2 years can be found as follows:

$$\begin{aligned} \text{months} &= \frac{\text{total days}}{\frac{\text{days}}{\text{month}}} \\ &= \frac{707 \text{ days}}{29.53059 \frac{\text{days}}{\text{month}}} \\ &= 23.941 \text{ months} \end{aligned}$$

YEAR 3

Since 355 days for the second month comes the closest to a whole month, the second year must be 355 days.

Year 1: Hebrew calendar lags the Gregorian calendar by 11 days
 356 days – 354 days = 11 days

Year 2: Hebrew calendar lags the Gregorian calendar by 10 days
 365 days – 355 days = 10 days

Total lag and the end of year 2: **21 days**

Because the Hebrew calendar is lagging by 21 days, the third year will need to be a 13 month year. All 13 months have 30 days. The length of 13 months can be:

$$\begin{aligned} 353 \text{ days} + 30 \text{ days} &= 383 \text{ days} \\ 354 \text{ days} + 30 \text{ days} &= 384 \text{ days} \\ 355 \text{ days} + 30 \text{ days} &= 385 \text{ days} \end{aligned}$$

Thus the possible lengths for a 13 month year are 383 days, 384 days, and 353 days.

As above the number of total months for year 3 can be calculated:

If it is **385 days** – then need to add 709 days from year 1 & 2 and 383 days from year 3

$$= 709 \text{ days} + 385 \text{ days}$$

$$= 1094 \text{ days in years 1, 2, 3.}$$

The number of months in years 1, 2, & 3 can be found as follows:

$$\begin{aligned} \text{months} &= \frac{\text{total days}}{\frac{\text{days}}{\text{month}}} \\ &= \frac{1094 \text{ days}}{29.53059 \frac{\text{days}}{\text{month}}} \\ &= 37.0397 \text{ months} \end{aligned}$$

If it is **384 days** – then need to add 709 days from year 1 & 2 and 384 days from year 3.

$$= 709 \text{ days} + 384 \text{ days}$$

$$= 1093 \text{ day in year 1, 2, 3.}$$

The number of months in years 1, 2, & 3 can be found as follows:

$$\begin{aligned} \text{months} &= \frac{\text{total days}}{\frac{\text{days}}{\text{month}}} \\ &= \frac{1093 \text{ days}}{29.53059 \frac{\text{days}}{\text{month}}} \\ &= 37.01246 \text{ months} \end{aligned}$$

If it is **384 days** – then need to add 709 days from year 1 & 2 and 384 days from year 3.

$$= 709 \text{ days} + 383 \text{ days}$$

$$= 1092 \text{ day in year 1, 2, 3.}$$

The number of months in years 1, 2, & 3 can be found as follows:

$$\begin{aligned} \text{months} &= \frac{\text{total days}}{\frac{\text{days}}{\text{month}}} \\ &= \frac{1092 \text{ days}}{29.53059 \frac{\text{days}}{\text{month}}} \\ &= 36.9786 \text{ months} \end{aligned}$$

The 384-day year is the closest number to 37 complete months. It is not chosen because it places the end of the month 2 days from completing the yearly cycle.

The 385-day year is chosen because it ends up one from completing that yearly cycle.

$$(0.0397 \text{ day} \times 24\text{hr/day} \approx 1\text{day})$$

Consider the following again:

Year 1(354 days):	Hebrew calendar lags the Gregorian calendar by	11 days
Year 2(355 days):	Hebrew calendar lags the Gregorian calendar by	10 days
Total lag and the end of year 2:		21 days

If **385 day** year is used:

Notice that the 355 days has a lag of 10 days. So at the end of 355 days in year 3 there would be a lag of 31 days. (**11days** + **10 days** + 10 days) If a 30 day month is added there is a lag of only 1 day.

If **384 day** year is used:

Notice that the 354 days has a lag of 11 days. So at the end of 354 days in year 3 there would be a lag of 32 days. (**11days** + **10 days** + 11 days) If a 30 day month is added there is a lag of 2 days.

If **383 day** year is used:

Notice that the 355 days has a lag of 12 days. So at the end of 353 days in year 3 there would be a lag of 33 days. (**11days** + **10 days** + 12 days) If a 30 day month is added there is a lag of 3 days.

Thus the 385 day year works out most evenly and Year 3 is 385 days.

YEAR 4

Year 4 can again be 353 days, 354 days, or 355 days long.

$$\begin{aligned} \text{If it is } \mathbf{353 \text{ days}} - \text{ then need to add } 1094 \text{ days from years 1, 2,} &= 1094 \text{ days} + 353 \text{ days} \\ \text{ \& 3.} &= 1447\text{days (years 1 - 4)} \end{aligned}$$

The number of months in the first 4 years can be found as follows:

$$\begin{aligned} \text{months} &= \frac{\text{total days}}{\frac{\text{days}}{\text{month}}} \\ &= \frac{1447 \text{ days}}{29.53059 \frac{\text{days}}{\text{month}}} \\ &= 49.000036 \text{ months} \end{aligned}$$

If it is **354 days** – then need to add 1094 days from years 1, 2, & 3. = 1094 days + 354 days
= 1448days (years 1 – 4)

The number of months in the first 4 years can be found as follows:

$$\begin{aligned} \text{months} &= \frac{\text{total days}}{\frac{\text{days}}{\text{month}}} \\ &= \frac{1448 \text{ days}}{29.53059 \frac{\text{days}}{\text{month}}} \\ &= 49.0339 \text{ months} \end{aligned}$$

If it is **355 days** – then need to add 1094 days from years 1, 2, & 3. = 1094 days + 355 days
= 1449days (years 1 – 4)

The number of months in the first 4 years can be found as follows:

$$\begin{aligned} \text{months} &= \frac{\text{total days}}{\frac{\text{days}}{\text{month}}} \\ &= \frac{1449 \text{ days}}{29.53059 \frac{\text{days}}{\text{month}}} \\ &= 49.0678 \text{ months} \end{aligned}$$

The 353 day year is the closest to a whole number of months. The problem is that by choosing that short year it reverts back to the same cycle of the first 4 years. Therefore the next closest year of 354 days is chosen. There are 1448 days to date.

YEAR 5

Year 5 can again be 353 days, 354 days, or 355 days long.

If a 353 day year there would be a total of 1801 days and 60.9876 days. Closest to whole month (61).

If a 354 day year there would be a total of 1802 days and 61.0214 days.

If a 355 day year there would be a total of 1803 days and 61.0553 days.

The 353 day year is the closest to the whole month (61) and thus is chosen.

Consulting Chart 6 it is found the total number of days in the first nineteen year cycle is 6939.

$$\begin{aligned}\text{months in 19 years} &= \frac{6969 \text{ days}}{29.53059 \frac{\text{days}}{\text{month}}} \\ &= 234.9766 \text{ months}\end{aligned}$$

This matches the travel of the moon matching the solar year. See chart 3 and look at the first 19 years. In column D the number 0 represents a whole day.

See Chart 6 for the complete 13 cycles, the number of days is found as 90216.

$$\begin{aligned}\text{months} &= \frac{90216 \text{ days}}{29.53059 \frac{\text{days}}{\text{month}}} \\ &= 3055.0016 \text{ months}\end{aligned}$$

This is the closest number with very little carry over to the next 13 cycles.